A THEORETICAL AND EXPERIMENTAL STUDY OF THE RESONANCE RADIOMETER

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BY

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By James Daniel Hardy

ABSTRACT

A galvanometer system for the measurement of very small voltages, of the order of 10^{-11} volts, is described and its application to the measurement of feeble radiations with a thermocouple discussed. It has been found that energies of the order of 1/200 of that previously detected by other devices could be measured with the Resonance Radiometer. The effect of the Brownian Movement on the system has been calculated and has been found to be about 1/200 that of the effect on the ordinary galvanometer.

An experiment is described for the measurement of the speed of action of thermocouples, and results on the splashed filament type are recorded.

From a consideration of the baseline fluctuations in the galvanometer curves published by Moll and Burger,[1] Ising[2] pointed out that the useful magnification of galvanometer deflections has a definite limit. These curves were reproductions from photographs made with the Moll "Thermo-relay" (a thermo-electric device for amplifying galvanometer deflections), and they exhibited a random zero shifting which the authors attributed to micro-seismic effects upon the suspended system. Using the theory previously developed by M. v. Smoluchowsky,[3] Ising showed that about two-thirds of this fluctuation could be accounted for as being due to the Brownian movement in the suspended system. This prediction was later confirmed experimentally by Ornstein and others,[4] who demonstrated the dependence of the fluctuations upon temperature.

The formula for the Brownian movement as first stated by Smoluchowsky is,

$$\tfrac{1}{2}A\overline{dx}^2 = \tfrac{1}{2}kT$$

where

A refers to the restoring force in a general meaning,
$\overline{dx}$ " " " root mean square displacement,
K " " " Boltzman gas constant,
T " " " Absolute temperature.

Using the above formula Ising calculated that for a single observation, the limit of sensitivity of the type galvanometer described by Moll is of the order of 10^{-9} volts. By means of the same formula it is easy

[1] W. S. H. Moll and H. C. Burger, Phil. Mag. *50*, 618; 1925.
[2] G. Ising, Phil. Mag. *1*, 827; 1926.
[3] M. v. Smoluchowsky, Phys. Zeit. *13*, 1069; 1912.
[4] L. S. Ornstein and Others, Proc. Roy. Soc. *115*, 391; 1927.

to show that the limit of voltage sensitivity of the best Leeds and Northrup galvanometer is of the same order of magnitude.

Since it is useless to magnify deflections which are not larger than the Brownian Fluctuations it appears that a natural sensitivity limit of fundamental character has been set upon the galvanometer as it is ordinarily used. Although the Brownian movement determines the best sensitivity that can be realized with a certain instrument, it is not, by any means, always possible to obtain such sensitivity. The usual situation is the one in which external disturbances are of greater importance than the Brownian Movement. From a consideration of this fact it occurred to Dr. A. H. Pfund[5] to apply the well known resonance principle in the hope of approaching the natural limit as closely as possible. The present paper is a report of the success of this method in not only approaching closely the natural limit of the galvanometer but in actually lowering the previous sensitivity limit by about two hundred times.

EXPERIMENTAL

The experimental arrangement, previously announced by Dr. Pfund is shown schematically in the following diagram.

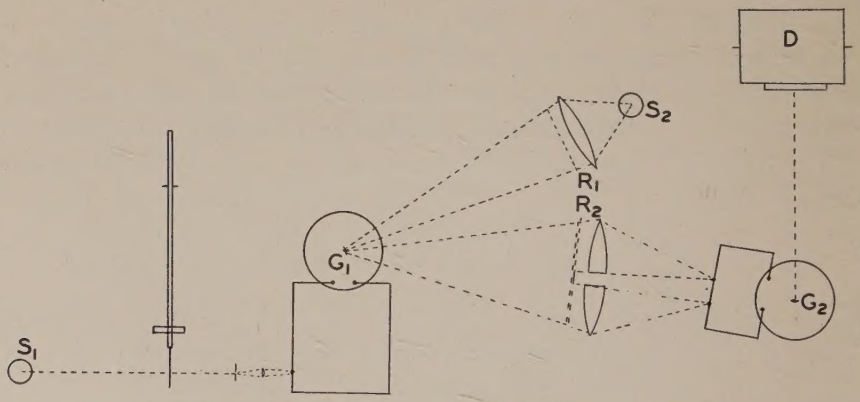

FIG. 1. *The experimental arrangement of the resonance radiometer.*

The light source S, was an ordinary automobile head-light bulb of which great care had been taken and whose calibration against a standard Hefner Lamp was frequently checked. This was the primary source of radiation which was to be measured, and variations in the intensity of the source were produced either by means of a rotating sector or by changing the current through the lamp. The lamp was usually burned

[5] A. H. Pfund, Science, *69*, 11; 1929.

at a current of 1.25 amperes which gave a radiation per unit area at one meter, of 0.013 that of a Hefner lamp. The light from this source was allowed to fall upon an aperture (cross section 0.075 sq. mm) one meter distant. It was then focussed by means of a glass lens upon a single junction thermocouple inside of an evacuated glass tube. Immediately in front of the apparatus was placed a magnetically controlled pendulum provided with a shutter so that the light from S_1 fell upon the thermocouple regularly every half period.

The period of the pendulum was adjusted so that it was exactly equal to that of the vibration galvanometer, G_1. The pulsating emf thus developed at the junction of the thermocouple was in resonance with the galvanometer and produced a resonance deflection. This resonance deflection was about twenty times as large as the steady deflection that would be produced by the same amount of radiation. The resonance deflections of galvanometer No. 1 were magnified by a tuned amplifier in the following manner:

The light from a second source S_2 was received by a convex lens and focussed upon the mirror of galvanometer No 1. The light after leaving the lens passed through a wire grating, R_1, spacing 2mm, so that upon reflection from the mirror of the galvanometer No 1 an image of R_1 was formed out in space at a distance of 40 cm. At this point was placed a second grating, R_2, identical with R_1, except that the middle bar was doubled. The doubling of this middle bar, threw one-half of R_2 out of step with the other; so that as the image of R_1 moved, due to a rotation of the galvanometer mirror, with respect to R_2, it caused a maximum of transmission on one side of the double bar and a minimum on the other.

The light from both halves of the second grid was received by the split convex lens and focussed separately upon the two junctions of a compensated thermocouple. This compensated thermo-couple was connected to second vibration galvanometer, G_2, of exactly the same period as the first, and the deflections of this galvanometer were read by photographing the deflections on a drum D.

The magnification obtainable with this amplifier could be made as large or as small as desired by changing the intensity of the Source S_2. In general a magnification from several hundred to two thousand was used with success.

The source of radiation, S_1, showed the following changes in intensity in a period of three months,

MAY 1929		AUGUST 1929	
Current	*Fraction of Hefner*	*Current*	*Fraction of Hefner*
2.2 Amps	0.27	2.2 Amps	0.27
1.75 "	0.13	1.75 "	0.12
1.50 "	0.05	1.50 "	0.05
1.35 "	0.014	1.25 "	0.013.

The lamp was used in connection with a storage cell; and no great variations in intensity were observed at any time while the lamp was burning under two amperes.

The pendulum was maintained at a constant amplitude by means of an electromagnetic device, and its period was as constant as necessary for this experiment.

The thermocouple was protected simply by the evacuated glass tube in which it was mounted and the tube was kept attached to an oil pump all the time. There were no attempts to shield the thermocouple or to protect it from external radiation, but even under these conditions very good results could be obtained.

An example of the relative sensitivity obtainable from this single junction thermocouple, in connection with a high sensitivity Leeds and Northrup galvanometer, and in connection with the resonance radiometer, might be of interest.

The thermocouple was connected in series with a sensitive (11 mm/micro-volt) Leeds and Northrup galvanometer and deflections read on a scale at three meters. The air currents through the room caused a zero unsteadiness of about 5 mm and it was not possible to judge deflections of much less than a centimeter. The thermocouple was then connected to the resonance radiometer, and under the same conditions, it was found possible to detect energy differences that would correspond to deflections on the Leeds and Northrup galvanometer of less than 0.01 mm.

The reason for this remarkable stability is clear when one considers that only 5 percent of the deflection of a stray radiation gets through galvanometer No 1, and due to the second tuned galvanometer in the amplifier only .2 percent of a stray deflection is actually recorded. So that under the same conditions of unsteadiness the resonance radiometer will be some 500 times as stable as a steady deflection instrument.

The thermocouple was of the type that has been made by Dr. Pfund for several years by means of splashing filaments upon a glass plate. The alloys used were Bismuth-Tin and Bismuth-Antimony and were fused together at the junction. No receiving junctions were attached.

In the process of fusing it was so arranged that the junction was the thinnest part of the element, thus making for rapid action. It was realized in the beginning that it would not be possible to use a vibration galvanometer of frequency greater than that of the reciprocal time of response of the thermocouple. There occur in the literature on the subject various values of the time taken for thermocouples to reach 99 percent of their temperature equilibrium value. Nicholson and Pettit have described a vacuum thermocouple which they claim reaches its equilibrium in about 6×10^{-3} seconds, and Moll claims about two seconds for his element; so it was decided to make a preliminary study of the speed of action of the type of thermocouple described above. It was at first noticed that with a vibration time of 1.7 seconds the thermocouple increased its effective sensitivity only 25 percent upon evacuation; while upon steady deflection under the same conditions it could be seen that the sensitivity of the thermocouple had been increased some six times. So, the following arrangement, really an Helmholtz Pendulum, was used to determine experimentally the time required for the thermocouple to reach the equilibrium point after exposure to radiation.

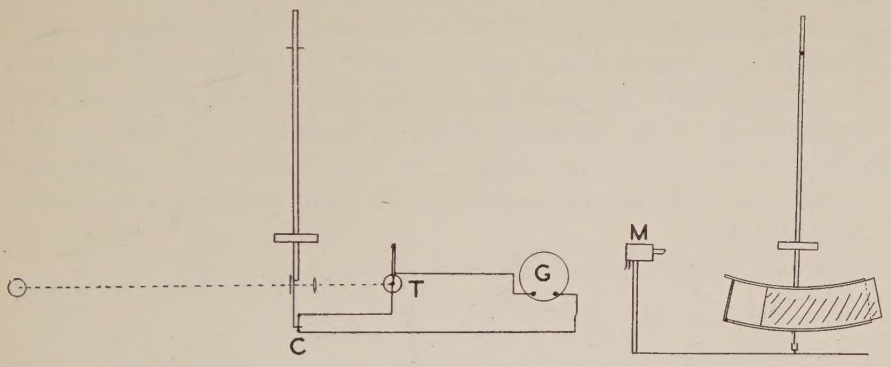

FIG. 2. *Schematic diagram of the arrangement used for measuring the time of response of thermocouples.*

The thermocouple T under investigation was connected in series with a long period, ballistic galvanometer, G, through a double contact key, C. A constant source of radiation was placed at some distance from the element and the energy brought to a focus upon the junction by means of a lens. A vacuum system was arranged so that pressure variations could be secured and their values read by means of a McLeod gauge. A pendulum was constructed with a shutter at the bottom and a projecting arm for throwing the double contact switch. The contacts were

arranged so that as the pendulum swung through the lowest part of its path it would close momentarily and then open the switch. The switch worked so easily mechanically and the pendulum was so heavy that this closing and opening of the galvanometer circuit produced practically no change in the motion of the pendulum. The interval during which the switch was closed was carefully measured by means of a chronograph and its value determined to be 0.065 seconds. This value varied slightly with the position of the sliding shutter, but its variation was measured and allowed for.

The sliding shutter at the bottom of the pendulum was the means used to vary the time of exposure of the thermocouple to the radiation. The time intervals between the exposing of the thermocouple to the radiation and the opening of the double contact key were also very carefully determined with the chronograph, and could be varied from 0.04 sec to 2.5 sec.

The pendulum, which was always dropped from the same height, was held aside by means of a small electro-magnet M. In this position the shutter cut off the radiation from the thermocouple. The double contact was adjusted so that the pendulum would close the galvanometer circuit long enough to give it a ballistic impulse. Various positions had been previously marked on the pendulum for the shutter, and the shutter was started with the shortest time of exposure. The pendulum was then released and the deflection of the galvanometer observed. As the position of the shutter was changed, to allow more and more time for exposure, the deflections increased up to a point where they reached a constant value.

Thus we have for the ballistic galvanometer,

$$D = \int_0^{\Delta t} i dt = \int_0^{\Delta t} f(t) dt$$

Where
- D is the deflection
- t is the time of exposure of the thermocouple to the radiation.
- Δt is the time during which the galvanometer is connected to the galvanometer
- i is the current
- e is the emf at the junction of the thermocouple.

For Δ t very small,

$$D \sim f(t)$$
$$\sim e$$

The values of t and D are determined experimentally, and it is then possible to plot the emf developed at the junction as a function of the time. The curves show the percentage response of the thermocouple, by which is meant the ratio of the observed deflection to the maximum deflection at equilibrium. Curve 1 shows that results obtained at atmospheric pressure; curve 2 at a pressure of 5×10^{-4} mm of mercury, and curve 3 at a pressure of 3×10^{-5}mm. The data on the thermocouple made with the splashed filaments, without receiver, are given in the following table.

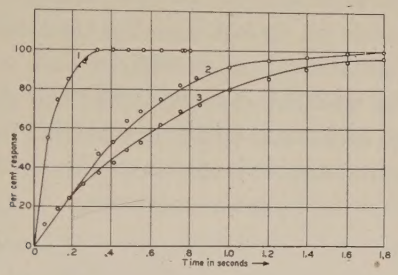

FIG. 3. *Curves showing results for the splashed filament type of thermocouple.*

Time of exposure	*Percentage response*		
	Pressure 1 Atm.	5×10^{-4}	3×10^{-5}
.07 seconds	55 percent	11.5 percent	11.5 percent
.13	75	19.5	19.5
.17	85	24.0	24.0
.24	95	31	30
.33	100	45	37
.41	100	53	42
.48		62·5	48
.55		69	53
.65		75	62
.75		82	68
.85		86	72
1.00		92	80
1.20		95	85
1.40		97	90
1.60		98	94
1.80		99	96
2.00		100	98
2.20		100	99

The increase in sensitivity obtained with the thermocouples upon evacuation was in the most cases between four and seven fold, and from the curves it can be seen with a full period of 4.5 seconds for the galva-

nometer, plenty of time is allowed for the thermocouple to come to equilibrium.

It might be well to state that while actually in use the pressure on the thermocouple was not so low as in the above experiment so that a two or three fold increase in sensitivity can be expected with the resonance radiometer upon further evacuation of the thermocouple.

After determining the period of the galvanometer the next task was that of constructing galvanometer No. 1 so that it would be sensitive, have a low moment of damping, or what is the same thing, a high resonance peak, and have as short a time as possible for coming to maximum amplitude. It can be shown that the time constant, or the time required to reach maximum amplitude, in case of good tuning, is, $T = I/f$, where I is the moment of inertia and f the moment of damping. Since it is necessary to have a small amount of damping to obtain a narrow, high resonance peak, it is also necessary to have a coil of small moment of inertia in order to get a small time constant.

Guided by the above considerations, galvanometers were constructed with varying sized coils until a time constant of about twenty-five seconds was obtained with a period of 4.5 seconds and a sensitivity greater than that of the Leeds and Northrup D'Arsonvals.

The dimensions of the galvanometer finally used were as follows: The magnets were of the usual horse-shoe type with a distance between poles of 25 mm. There was no radial field center piece and the intensity of the magnetic field could be further decreased by means of a magnetic shunt. The coil was constructed in this laboratory and made of ordinary copper wire with silk insulation (No. 36 B and S guage). The number of turns used was four, and the size of the coil was 8×40 mm. A glass capillary was used as an axis and upon it was mounted a very small concave mirror. The coil when complete had a moment of inertia of .0261 gm cm^2, and a resistance of less than one ohm. The suspensions were of .0007 Cu strip making the total resistance of the galvanometer about five ohms. No care was taken in the selection of the wire and the instrument was distinctly of the homemade type, despite the rather high sensitivity, 12 mm per micro-volt.

A preliminary experiment showed that upon evacuation of the galvanometer an increase in sensitivity of nearly twofold could be expected. Unfortunately no further experiments have been made in that direction. Also a careful selection of magnetically free wire should result in an increase in sensitivity.

The galvanometer was connected in series with the thermocouple under the conditions stated above, except that the aperture was removed. The pendulum was set in motion and the tuning proceeded as follows:

The period of the pendulum was adjusted so that it was exactly equal to that of the first galvanometer by observing the variations in amplitude with time. The first curve in Fig. 4 shows the effect of bad tuning; the second curve, close tuning; and the third curve, exact tuning. Very good tuning could be obtained by this method, and once adjusted the instrument would remain in tune for several days. As soon as the pendulum had been adjusted to the period of the first galvanometer, the second galvanometer was connected to the first thermocouple and its period made equal to that of the pendulum, using the above method, by changing the length of the upper suspension of the coil. In this manner the two galvanometers and the pendulum were made to be accurately in tune.

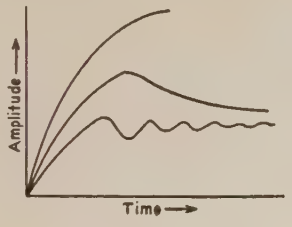

FIG. 4. *Curves showing the effect of tuning upon the amplitude.*

These curves are what would naturally be expected from a system having a damped and undamped vibration superimposed upon it. If the coil is too heavily damped, the above phenomena cannot be easily observed

After having constructed a suitable galvanometer it was necessary to find a place sufficiently free of mechanical disturbances so as to allow the application of the thermoelectric amplifier designed by Dr. Pfund Many types of vibration-free supports were tried, including that suggested by W. A. Parlin[6] and that suggested by Johnsrud,[7] but neither gave the desired stability. So at last the instrument was moved down into the basement of the Physics Building and mounted on a shelf on the wall. All work was done late at night. Under these conditions a stability was obtained such that under a magnification of 2000 times

[6] W. A. Parlin, Phys. Rev. *34*, 81; 1929.
[7] A. L. Johnsrud, J. Opt. Soc. America, *10*, 609; 1925.

the zero amplitude was only five millimeters or less.

The thermoelectric amplifier was mounted on a table out of contact with the galvanometer and the wall so that adjustments could be made without disturbing the galvanometer. The second galvanometer, identical with the first, except with a method of tuning added, was connected to the amplifier. This galvanometer, No. 2, had been previously carefully tuned to the period of galvanometer No 1. The system thus adjusted was ready for use, and the deflections of galvanometer No. 2 were recorded photographically.

RESULTS

After the instrument had been mounted upon a sufficiently stable support measurements were carried out to determine the smallest possible amount of energy that could be detected. The small automobile headlight was used as a standard light, and an aperture .3 mm in diameter, a meter away, was used to determine the size of the cone of rays admitted to the thermocouple.

The first experiment was performed with the lamp burning at 1.25 amperes, and the light energy reduced by means of a rotating sector. The deflections were recorded photographically and ten deflections on each value were read. If the radiant energy, at transmision 100 percent passing through the aperture, be reduced to the energy received by a sq. millimeter area, one meter distant, from a Hefner lamp, it would correspond to .0011 of the energy.

Transmission	*Observed*	*Deviation*	*Calculated*
100 percent	11.1 cm	.08	—
50	5.6	.08	5.57
45	5.1	.02	5.0
35	3.9	.10	3.9
30	3.4	.10	3.3
25	2.8	.10	2.8
20	2.2	.05	2.2
15	1.8	.05	1.7
10	0.9	.05	1.1

The third column gives the average deviation from the mean for the several readings recorded under each transmission value. It can be seen that a change of five percent of .0011 Hefner units can be easily recorded.

The sensitivity of the Leeds and Northrup galvanometer had been previously determined to be 11 mm/microvolt with the scale at one meter. It was found that the resonance radiometer responded vigorously

to radiations that could not be detected by the Leeds and Northrup galvanometer with the scale at five meters. Thus in order to make a direct comparison, in terms of volts, it was necessary to remove the .075 mm^2 aperture and replace it with one of 2 mm^2 area. Assuming, then, that the energy received was proportional to the aperture area, it was calculated that the 5 percent change mentioned above corresponded

FIG. 5. *Sample of photographically recorded deflections.*

to a deflection on the Leeds and Northrup galvanometer of 0.001 mm scale at 3 meters, or to a voltage of 4×10^{-11} volts, a voltage of the order of 1/200 that heretofore measurable.

Further experiments with sensitive thermocouples have led to the conclusion that the smallest energy difference detectable with the resonance radiometer, under good conditions, is of the order of 5×10^{-6} ergs/sec.

THEORY

The problem of describing the Brownian Movement in such a system consists of developing the irregular force into a Fourier Series, solving the equation of motion and summing up over all frequencies with the proper regard for selectivity of the amplifier. A very similar problem has been attacked by Hull and Williams[8] in the case of the shot-effect,

[8] A. W. Hull and N. H. Williams, Phys. Rev. *25*, 147; 1925.

and also by Uhlenbeck and Goudsmit[9] for the case of the Brownian Movement of a suspended mirror. In the calculation below material has been drawn from these two sources in both argument and procedure.

The 'k-th' component of the Brownian Fluctuations in galvanometer No. 1, according to Uhlenbeck and Goudsmit,[(9)] will produce the mean square amplitude given by,

$$\bar{\Phi}_k^2 = \frac{2}{\tau} \frac{\overline{T(t)^2}\Delta t}{(\omega^2 - \omega_k^2) + \gamma^2\omega_k^2} \tag{1}$$

Where $\overline{T(t)^2}$ represents the mean square of the periodic force due to the Brownian Movement devided by the moment of inertia; t represents the time; ω represents the free period of the galvanometer; ω_k represents the frequency of the k-th component of the Brownian Fluctuation; γ represents the frictional forces divided by the moment of inertia of the coil; τ time of observation.

It has been shown that[9]

$$\tfrac{1}{2}D\bar{\phi}^2 = \tfrac{1}{2}D\sum_0^\infty \bar{\phi}_k^2 = \tfrac{1}{2}kT \qquad \begin{matrix} k = \text{gas constant} \\ T = \text{absolute temp.} \end{matrix} \tag{2}$$

Substituting this in 1 we have

$$\left\{\frac{2}{\tau}\overline{T(t)^2}\Delta t\right\}\frac{1}{2}D\sum_0^\infty \frac{1}{(\omega^2 - w_k^2)^2 + \gamma^2\omega_k^2} = \frac{1}{2}kT \tag{3}$$

$$\sum_0^\infty \frac{1}{(\omega^2 - \omega_k^2)^2 + \gamma^2\omega_k^2} \leqq \frac{t}{2\pi\omega^3}\int_0^\infty \frac{dx}{(1 - x^2)^2 + \left(\frac{\gamma^2}{\omega^2}\right)x^2}$$

where

$$x = \frac{\omega_k}{\omega} = \frac{2\pi k}{\tau\omega}$$

$$= \frac{\pi\omega}{2\gamma}\left(\frac{\tau}{2\pi\omega^3}\right) = \frac{\tau}{4\gamma\omega^2}.$$

Then substituting, we find for $\overline{T(t)^2}$

$$\overline{T(t)^2} = \frac{2kT\gamma\omega^2}{D\cdot\Delta t}. \tag{4}$$

Putting this value in 1 we get

[9] G. E. Uhlenbeck and S. Goudsmit, Phys. Rev. *34*, 145; 1929.

$$\bar{\phi}_k^2 = \frac{2}{\tau} \cdot \frac{2kT\gamma\omega^2}{D} \cdot \frac{1}{(\omega^2 - \phi_k^2)^2 + \gamma^2\omega_k^2}. \tag{5}$$

If the amplifying system is attacked we have (following Hull and Williams),[8]

$$\frac{\bar{\Phi}^2}{\bar{\phi}^2} = A_0^2 f_i(x) \qquad x = \frac{\omega_k}{\omega}.$$

Let $A_0 = 1$, then

$$\bar{\Phi}^2 = \frac{4kT\gamma\omega^2}{\tau D} \sum_0^\infty \frac{f\left(\frac{\omega_k}{\omega}\right)}{(\omega^2 - \omega_k^2)^2 + \gamma^2\omega_k^2} \tag{6}$$

Φ = mean square amplified amplitude.

A_0 = maximum value of amplification

$f(x)$ = represents the amplifying curve whose max. value = unity.

Equation 6 gives the mean square amplitude after passing through the amplifier.

$$\sum_0^\infty \frac{f\left(\frac{\omega_k}{\omega}\right)}{(\omega^2 - \omega_k^2)^2 + \gamma^2\omega_k^2} \cong \frac{\tau}{2\pi\omega^3} \int_0^\infty \frac{f(x)dx}{(1 - x^2) + p^2x^2} \qquad p = \frac{\gamma}{\omega} \tag{7}$$

If $f(\omega_k)/\omega = 1$ then we have

$\bar{\Phi}^2 = kT/D$ since the value of the integral would be $\pi\omega/2\gamma$. (8)

But if $f(x)$ is variable we define the amplification factor F,

$$F = \int_0^\infty \frac{f(x)dx}{(1 - x^2) + p^2x^2} \Big/ \frac{\pi\omega}{2\gamma}. \tag{9}$$

Then

$$\bar{\Phi}^2 = kTF/D \text{ after amplification.} \tag{10}$$

To find a value for F it is necessary to solve the integral in 9, and this is done by a method due to Schottky.[10]

Since only a small range of values of x contribute greatly to the value of the integral, we set with sufficient approximation.

$$\int_0^\infty \frac{f(x)dx}{(1 - x^2) + p^2x^2} = \int_{1-a}^{1+a} \frac{f(x)dx}{(1 - x^2)^2 + p^2x^2}; \quad a \ll 1.$$

If we add to the given integral another integral

[10] I. Schottky, An. d. Phys. *57*, 556; 1918.

$$\int_{1-a}^{1+a} \frac{xf(x)dx}{(1-x^2)^2+p^2x^2} \text{ formed by making } z=1/x$$

in the original integral and finally writing x for z the limits of the new integral will be the same as those of the old if $a^2 \ll 1$, since $1/1-a=1+a$, and $1/(1-a)^2=1$.

Also, $f(1/1-\delta)=f(1+\delta)=f(1-\delta)$. Therefore

$$\int_{1-a}^{1+a} \frac{f(x)dx}{(1-x^2)^2+p^2x^2} = \frac{1}{2}\int_{1-a}^{1+a} \frac{(1+x^2)f(x)dx}{(1-x^2)^2+p^2x^2}. \tag{11}$$

Making a change of variable such that

$$y = \frac{1}{p}\left(x - \frac{1}{x}\right)$$

we get

$$\frac{\pi F}{2p} = \frac{1}{2p}\int_{-2a/p}^{+2a/p} \frac{\phi(y)dy}{1+y^2}, \text{ where } \phi(y) = f(x). \tag{12}$$

Replacing the integral by a summation,

$$F = \frac{1}{\pi} \sum_{-2a/p}^{2a/p} \phi(y_n)\frac{\delta y}{1+y^2}$$

$$F = \frac{2}{\pi} \sum_{0}^{2a/p} \phi(y_n)(\tan^{-1} y_n - \tan^{-1} y_{n-1}). \tag{13}$$

The steps $y_n = 2n\delta x/p$ were taken at intervals of $\delta x = .08$ and ten steps were found to be sufficient.

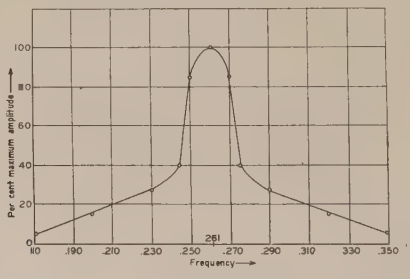

FIG. 6. *Resonance curve for the vibration galvanometers.*

In order to carry out the above summation it is necessary to have the resonance curve of the amplifier. This curve is shown in Fig. 6. It is also necessary to know the value of f, I, and ω. All these values were determined experimentally and the summation carried through in

exactly the same manner as the one described in Hull's paper.[8] The value of F thus determined is 0.826

We have, then, finally from equation 10, for the particular amplifier

$$\overline{\Phi} = \sqrt{\frac{.826kT}{D}}. \tag{14}$$

In order to determine the low voltage limit for such a system it will be of use to deduce the formula for the voltage sensitivity of a vibration galvanometer.[11]

The equation of motion of the moving system is, as before,

$I\ddot{} + f\dot{\phi} + D\phi = Bi \qquad i = J' \cos p\,t$

Solving we get

$$\phi = \frac{BJ\sqrt{2}}{\sqrt{p^2f^2 + I^2(p_0^2 - p^2)^2}}. \tag{15}$$

$p_0 = 2\pi\nu_0$

$p = 2\pi\nu$

$r =$ resistance of circuit.

But it can be shown that the retarding torque in the case of voltage indications is $\frac{rf+B^2}{r}\frac{d\phi}{dt}$ instead of $f\ d\phi/dt$. Therefore letting $p = p_0$ and substituting for f^2 in 15 we have

$$\phi = \frac{BE\sqrt{2}}{p_0(rf + B^2)} \quad \text{or} \quad \frac{\phi}{E} = S_v \tag{16}$$

$$S_v = \frac{B\sqrt{2}}{p_0(rf + B^2)} \text{ voltage sensitivity.} \tag{17}$$

If 17 is differentiated with respect to B and set equal to zero it will be found that

$$rf = B^2 \tag{18}$$

for the condition of maximum sensitivity. As determined experimentally, in the case of galvanometer No. 1, we have a direct current sensibility $= 2 \times 10^4\, B/D$

$= 10$ mm/micro amp.

$D = .0684$

[11] F. W. Wenner, Bull. Bureau Stand. *6*, 347; 1909.

$$\therefore B = 3.22 \times 10^3$$
$$r = 20 \times 10^9 \text{ c.g.s. units}$$
$$f = 6.4 \times 10^{-4}.$$

Then $B^2 = 1.16 \times 10^7$

$$rf = 1.28 \times 10^7$$

Which shows that the adjustment of magnetic forces, etc., has been such as to give a sensitivity near to the maximum.

Substituting equation 18 into equation 17 we get for the best sensitivity,

$$S_\nu = \frac{1}{\sqrt{2p_0 B}} = \frac{1}{p_0}\sqrt{\frac{1}{2rf}} = \frac{\delta\phi}{\delta E}. \tag{19}$$

Now we assume that to detect a single deflection, the provoked amplitude must be twice that due to the Brownian Fluctuations or

$$\delta\phi = \overline{2\delta\phi}. \tag{20}$$

By substituting this value in equation 19 we get that the minimum observable voltage on one single deflection, with the introduction of the practical units, is

$$(\delta\nu')_{\min} = 7.20 \times 10^{-10}\sqrt{\frac{\nu_0^2 r' f}{D}} \text{ volts} \tag{21}$$

where r' = resistance in ohms.

This equation shows that by decreasing the damping coefficient f it is possible to decrease very much the effective Brownian Fluctuations. In the present apparatus

$$\nu_0 = .261 \qquad f = 6.32 \times 10^{-4}$$
$$r' = 5 \qquad D = .0684$$

or

$$(\delta\nu')_{\min} = 3.75 \times 10^{-11} \text{ volts.}$$

Physically the above calculation amounts to this.[12] We have supposed the distribution of energy throughout the spectrum of the Brownian Movement to be uniform. The impressing of this energy upon the tuned galvanometer No. 1 would result in distorting the Brownian Movement spectrum into a peak. (Fig. 7.)

The maximum of the curve will lie at the resonance frequency of the galvanometer and the width of the band will depend upon the damping of the galvanometer. The energy absorbed from the spectrum of the Brownian Movement will be proportional to the area under the reso-

[12] K. Herzfeld, kindly suggested by him.

nance band and this area is independent of the damping. Since as the damping is increased the peak is lowered and at the same time broadened, and vice-versa. (This means that $D\bar{\phi}^2$ is independent of the damping.)

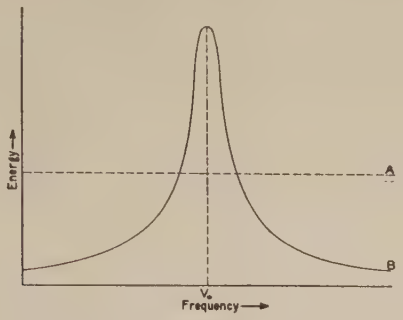

FIG. 7. *Distribution of brownian movement energy with frequency.*
A. *Actual distribution of brownian movement energy.*
B. *Distribution of energy in galvanometer No. 1.*

The energy impressed upon the instrument by means of the pendulum will also have a spectrum* because it will not be possible to make the pendulum absolutely isochronous. That is, the frequency of the pendulum will vary over a very narrow but finite range. Experimentally, of course, the width of the resonance band of the galvanometer will always be much wider than that of the pendulum. Thus the superposition of the energy from the source upon that of the already existing Brownian Movement would result in the following curve.

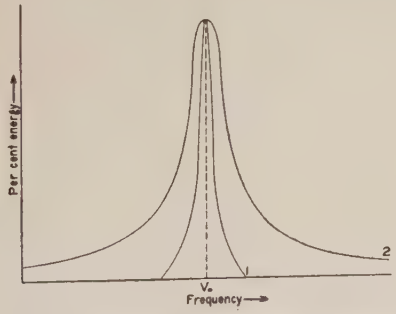

FIG. 8. *1. Resonance band of pendulum. 2. Brownian movement in galvanometer No. 1.*

It can be seen that decreasing the width of the galvanometer band will not cut out anything from the width of the pendulum spectrum, so

* The fact that the pendulum has a resonance band of finite width has not been considered in the calculation.

that the increase in the height of the peak will increase the effect of the external source and will result in a relative decrease in the effect of the Brownian Movement. As soon, however, as the width of the galvanometer band has been decreased until it is of the same width as that of the pendulum the limit of useful narrowing has been reached. Since a further narrowing would cut out from the external source at the same rate as from the Brownian Movement.

There are two methods for decreasing the band of galvanometer No. 1: The first is that of decreasing the damping. Decreasing the damping is limited by the fact that the time required from the instrument to record maximum amplitude after being excited becomes longer as the damping becomes smaller. But by adjusting the moment of inertia of the coil it is possible, as has been shown, to make the band so narrow that it decreases the influence of Brownian movement, relatively, over two hundred times.

The second method has to do with the application of the tuned amplifier.

Only the case discussed above shall be considered, that is, where the spectrum of the pendulum (curve 1 in the following figures) is narrow compared with the spectrum (due to damping) of the first resonance galvanometer (curve 2 in the following figures). The effect of the width of resonance on the amplifier (curve 3) is as follows: (a) The width of the band of the second galvanometer much *greater* than that of the first.

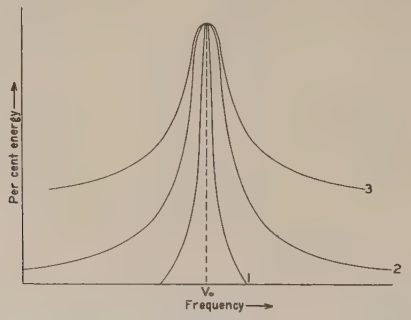

FIG. 9. *Showing the relative widths of galvanometer and pendulum resonance bands in the case where the effect of the second galvanometer in decreasing the effect of the brownian movement is very small.*

Curve 1, pendulum band.
Curve 2, band of galvanometer No. 1.
Curve 3, band of galvanometer No. 2.

In this case since the spectrum of the Brownian Movement (curve 2) is amplified in the same manner as the pendulum spectrum no advantage

occurs. (b). The damping in the second galvanometer is *equal to* that in the first. The pendulum system is magnified without anything being cut out, due to its narrowness. The Brownian Movement spectrum is cut down somewhat (factor 0.82) due to the fact that in the lower, broader parts of curve 2 the amplification is less than at the peak. The improvement is therefore 1/0.826 times. This is the experimental case used above.

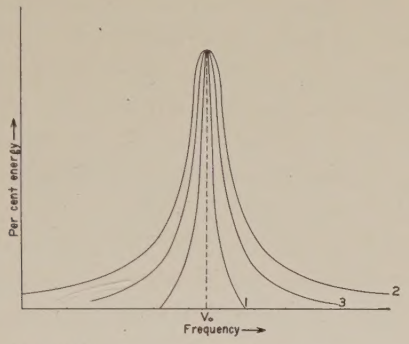

FIG. 10. *Showing the widths of the resonance bands in the case where galvanometer No. 2 is most effective in reducing the brownian movement.*

(c). Still more advantageous would be a galvanometer having a spectrum between curves 1 and 2, which would amplify the pendulum spectrum 1 unmodified but would cut down the Brownian movement spectrum 2. Unfortunately this cannot be done because then the galvanometer would have to have so small a damping that it would take too long to reach its full amplitude.

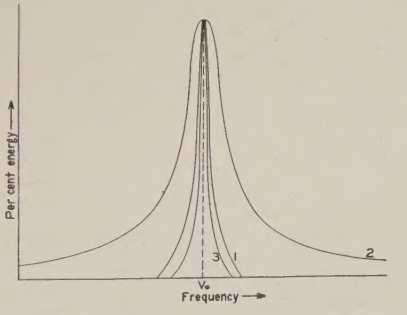

FIG. 11. *Showing the case in which the effect of the second galvanometer is zero.*

(d). If the curve 3 would lie within 1, no further advantage would accrue, as the pendulum spectrum, 1 and 2 could be cut down alike.

To sum up; it follows that the amplifier works more advantageously,

the closer its resonance band has the form of the spectrum that is to be measured, provided it is not narrower.

It is interesting to note in this connection that the amplifying system not only amplifies but cuts out from the effects of stray disturbances and from the Brownian Movement.

The effect of the whole system is to cut down the effect of the Brownian Movement and stray radiations about 500 times.

Experience with the Resonance Radiometer convinces one of the great sensitivity that can be realized with such an instrument. Due to the fact that it requires some thirty seconds for the instrument to give maximum response to radiation it is not thought that it will be useful for any except the most delicate work. But it is felt that the field of high sensitivity measurements, with the use of such an instrument can be greatly extended.

In conclusion I wish to acknowledge my indebtedness to Dr. A. H. Pfund upon whose suggestion the above work was undertaken, and to Dr. K. F. Herzfeld for his assistance in matters of theory.

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VITA

James Daniel Hardy was born in Georgetown, Texas, August 11, 1904. He received his preparatory education at the Gulf Coast Military Academy and the Gulfport High School; he completed his college education at the University of Mississippi, taking the A.B. degree in 1924 and the A.M. degree in 1925, and was appointed Assistant Professor of Physics in the University of Mississippi in 1925. He entered The Johns Hopkins University in the fall of 1927 and pursued courses in Mathematics under Professors Murnaghan and Cohen, in Physics under Professors Wood, Herzfeld, Pfund and Hubbard. He wishes to express his appreciation to Professors Hubbard, Murnaghan and Wood for their inspiration and encouragement, and especially to Professors Pfund and Herzfeld without whose assistance this work could not have been carried out.